Discovering the Tarski Boundary

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The Tarski Boundary (name suggested to us by Ali Enayat) is the line demarcating conservative extensions of basic compositional theory of truth (CT^- over PA) from the non-conservative ones. There are some principles that are well-known to lie on the non-conservative side: the Global Reflection Principle over PA, i.e. the following sentence

$$\forall \phi \in \mathcal{L}_{\mathrm{PA}} \big(\mathrm{Pr}_{\mathrm{PA}}(\phi) \rightarrow T(\phi) \big)$$

is one of the most well-known. In the paper "Bounded Induction and Satisfaction Classes" (1986) Henryk Kotlarski proved that (over $\mathrm{CT}^-(\mathrm{PA})$) the above principle is equivalent to the Δ_0-induction schema for formulae containing the truth predicate (denote this theory by CT_0). In 2008 Heck and Visser (independently) spotted a mistake in his proof: it was not clear how to justify that provably in CT_0 all logical axioms are true. In our talk we show how to fix this gap. Moreover we will prove that CT_0 admits various interesting axiomatizations, including

1. $\mathrm{CT}^-(\mathrm{PA})+$ Global Reflection Principle over PA
2. $\mathrm{CT}^-(\mathrm{PA})+$ "All first order tautologies are true"
3. $\mathrm{CT}^-(\mathrm{PA})+$ "Truth is closed under provability in first order logic"
4. $\mathrm{CT}^-(\mathrm{PA})+$ "Truth is closed under provability in propositional logic"
5. $\mathrm{CT}^-(\mathrm{PA})+$ "All axioms of PA are true" $+$ "Disjunction of finitely many sentences is true if and only if one of these sentences is true"

The equivalence of first four theories was proved by Cezary Cieśliński and the fact that the last one proves Δ_0-induction has been recently shown by Ali Enayat. The above result show that all the above extensions of $\mathrm{CT}^-(\mathrm{PA})$ lies on the non-conservative side.